

CC-I

MATHEMATICAL PHYSICS-I

1 MARK QUESTIONS:

1. The equation $ax^2 + bx + c = 0$ represents equations of _____
2. In the equation $y = x^2 - 6x + 8$, the coordinates of the vertex are ____ & ____
3. A function $y = f(x)$ is continuous at a point if its graph has _____ at that points.
4. The function $f(x) = [x]$ is _____ at all integers.
5. If $xy + x^2y^2 = \text{constant}$, then $\frac{dy}{dx}$ is ____
6. The order of the equation $\frac{dy}{dx} + y \tan x = \sin 2x$.
7. A function $f(x,y)$ is called homogeneous of degree n if $f(tx, ty)$ _____
8. Every polynomial is continuous at every point of the real line. True or False.
9. If $f(x)$ is differentiable at every point of its domain then it must be continuous in that do but the converse is not true. Do you agree with this statement?
10. One of the integrating factors of the equation $-ydx + xdy = 0$ is _____
11. If the first order differential equation is not exact then it can be made exact by multiplying it with a quantity known as _____
12. Does vector product of two vectors produce a vector?
13. What is the projection of A along B?
14. The value of scalar product is _____ under rotation.
15. If the coordinate surfaces are mutually perpendicular to each other, then they are called _____ system.
16. If the curvilinear coordinate surfaces $u = \text{constant}$, $v = \text{constant}$, $w = \text{constant}$ intersect at right angles then the curvilinear coordinate system is known as _____ system of coordinates.
17. The expression for the are length ds in terms of h_1 , h_2 and h_3 is given by $ds^2 =$ _____.
18. The cylindrical coordinates of point P in space are represented as _____
19. The spherical polar coordinates of point P in space are represented as _____.
20. Write the expression for velocity in cylindrical coordinate system.

1.5 MARK QUESTIONS:

1. Solve $(x^2 + y^2) - 2xy \, dy = 0$
2. Find unit vector perpendicular to each of vectors $\vec{A} = 2\hat{i} + -\hat{j} + \hat{k}$ and $\vec{B} = 3\hat{i} + 4\hat{j} - \hat{k}$.
3. Find the constant P for which $\vec{A} \times \vec{B} = \vec{C}$ Where $\vec{A} = \hat{i} + 2\hat{k}, \vec{B} = \hat{i} + P\hat{j} - \hat{k}$ and $\vec{C} = -2\hat{i} + 3\hat{j} + \hat{k}$
4. Define curl of a vector in Cartesian Co-ordinates system.
5. Find the Laplacian in Cartesian Co-ordinate system.
6. Show that $\nabla^2 \cdot (r^n \vec{r})$ vanishes for $n = -3$.
7. Determine curl of \vec{A} if $\vec{A} = x^2\hat{i} - y^2\hat{j}$.

8. Evaluate $\nabla^2 \left(\frac{1}{r} \right)$.
9. Prove that $f(x) \delta(x - a) = f(a) \delta(x - a)$.

10. Define surface integral and explain why it is called a flux.
11. Define volume integral with its physical significance.

12. Prove $\delta(ax) = \frac{1}{|a|} \delta(a - b)$.

13. Prove $\int_{-\infty}^{\infty} \delta(x - a) \delta(x - b) dx = \delta(a - b)$.

14. If \vec{a} is a constant vector, then prove $\vec{\nabla} \times (\vec{a} \times \vec{r}) = 2\vec{a}$.

15. If $\vec{A}(t)$ is a vector of constant magnitude then prove $\vec{A} \cdot \frac{d\vec{A}}{dt} = 0$.

16. Find $\vec{\nabla} \left(\frac{1}{r} \right)$.

17. The equation of a surface is given by $2x^2 + y^2 - 4z^2 + 3 = 0$, find unit vector perpendicular to the surface at $(1, 1, 1)$.

18. For polar Co-ordinates $x = r \cos\theta, y = r \sin\theta$ the prove $\frac{\partial(x, y)}{\partial(r, \theta)} = r$

19. Prove, $\int_S \vec{r} \cdot d\vec{s} = 3V$, V is the volume enclosed by surface 'S' and \vec{r} is the position vector.

2.5 MARK QUESTIONS:

$$\int_C \vec{r} \cdot d\vec{r} = 0$$

1. Solve
2. Explain Lagrange multipliers.
3. Explain properties of vector rotation.
4. State and explain properties of Dirac-Delta function.
5. Find the curl of a vector field \vec{v} .
6. Express Laplacian in Cylindrical Co-ordinates.
7. Explain about Jacobian.
8. Give one application of Stokes theorem
9. Find the value of $\int_{-\infty}^{\infty} x(\delta(x-4))dx$.
10. Define curl of vector.
11. Give the notation of infinitesimal volume integral.

5 MARK QUESTIONS:

1. Prove that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$
2. Prove that $\vec{A} \times (\vec{B} \times \vec{C}) + \vec{B} \times (\vec{C} \times \vec{A}) + \vec{C} \times (\vec{A} \times \vec{B}) = 0$
3. Show that the scalar product is invariant under rotation.
4. Prove $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A}$
5. State and prove Gauss's divergence theorem.
6. State and prove Stoke's theorem.
7. Evaluate $\int_{\frac{-1}{2}}^1 \int_{-x}^{1+x} (x^2 + y) dx dy$

8. Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = x\hat{i} + xy\hat{j}$ and C is the boundary of square in plane $Z=0$ bounded by lines $x=0, y=0$ and $x=1$ and $y=1$.
9. Evaluate $\oint_C (-ydx + xdy)$ of 'c' is the circumference of the circle $x^2 + y^2 = 1$
10. Express the cylindrical co-ordinates (ρ, ϕ, z) in terms of Cartesian co-ordinates (x, y, z) and vice versa.
11. A particle is moving in space. Find its position vectors, velocity and acceleration in terms of spherical polar co-ordinates.
12. Find $\vec{\nabla} \times \vec{A}$ for cylindrical co-ordinate system.
13. Find $\vec{\nabla} \times \vec{A}$ in spherical polar co-ordinate system.
14. Express divergence of vector point function in spherical co-ordinate system.
15. Derive expression for velocity and acceleration in cylindrical co-ordinate system.
16. Derive Laplacian in spherical polar co-ordinate system.
17. Define gradient of a scalar field and give its geometrical interpretation.
18. If $M(x, y)dx + N(x, y)dy = 0$ is not exact and has a general solution $F(x, y) = C$ then prove that there exists an integrating factor.
19. Find the integrating factor of $xdy - ydx = 0$ and solve the equation in each case.
20. Find minimum value of $x^2 + y^2 + z^2$ the subject to condition by Lagrange's method.