<u>(Unit- 1)</u>

Matrices and Determinants

1. Solve the following system of equations using the matrix method :

(i) $2x-3y = 1$
$x+ 5y = 7$

(ii) $3x+ 2y = 6$
$5x+ 4y = 8$

(iii) $4x-3y = 5$
$3x-5y = 1$

(iv) $2x-y = 13$
$-x+2y=-11$

(v) $2x-y + 2 = 0$
$3x + 4y-3 = 0$

2. Solve by the matrix method:

(i) $x-y+z= 4$
$2x + y-3z = 0$
$x + y + z = 2$

(ii) $x+ y+ z = 2$
$2x + 2y + 3z = 7$
$5y-z + 13 = 0$

(iii) $8x + 4y + 3z = 13$
$2x+y+z= 5$
$x + 2y + z = 5$

(iv) $2x + 2y + z = 13$
$4y + z = 17$
$-3x + 2y = 3$

(v) $x+ y+ z = 3$
$2x-y+z = 2$
$x-2y+3z = 2$

(vi) $3x + 2y+ 4z = 19$
$2x-y + z = 3$
$6x+ 7y-z= 17$

3. Solve the following equations by using matrices :

$4x + 3y + 2z + 7 = 0$
$2x +y-4z+1 =0$
$x-7v-2=0$

4. Solve the following equations by the matrix method :

(i) $4,x + 3y + = 8$
$2x + y + 4z = -4$
$3x + z = 1$

(ii) $x + y + 2z = 4$
$2x-y + 3z= 9$
$3x-y-z = 2$

5. An amount of Rs. 5000 is put into three investments at the rates of interest of 6%, 7% and 8% per annum respectively. The total annual income is Rs. 358. If the combined income from the first two investments is 70 more than the income from the third, find the amount of each investment by using matrix algebra.

6. Mr. X has invested a part of his investment in 10% bond A and a part in 15% bond B. His interest income during first year is 4000. If he invests 20% more in 10% bond A and 10% more in 15% bond B, his income during second year increases by Rs. 500. Find his initial investment and the new investment in bonds A and B using matrix method

7. A salesman has the following record of sales during three months for three items A, B and C which have different rates of commission.

Months	Sales			Total Commission drawn (in Rs.)
	A	B	C	
January	90	100	20	800
February	130	50	40	950
March	60	100	30	850

Find out the rates of commission on items A, B and C.

8. An automobile company uses three types of steel Si, S2 and S3 for producing three type of cars CI , C2 and C3. Steel requirements (in tons) for each type of cars are given below :

Steel		Cars		
		C1	C2	C3
	S1	2	3	4
	S2	1	1	2
	S3	3	2	1

Determine the number of cars of each types which can be produced using 29, 13, and 16 tones of steels of three types respectively.

9. A firm produces two products 131 and P2 passing through two machines M1 and M2 before completion. MI can produce either 10 units of PI or 15 units of P2 per hour. M2 can produce 15 units of either product per hour. Find daily production of PI and P2 if time available is 12 hours on MI and I0 hour on M2 per day using matrix inversion.

10. Find the transpose of each of the following matrices :

(i) $A = \begin{pmatrix} 2 & 5 \\ 6 & 15 \end{pmatrix}$

(ii) $B = \begin{pmatrix} 1 & -2 & 3 \\ 0 & 2 & -1 \\ -4 & 5 & 2 \end{pmatrix}$

(iii) $C = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{pmatrix}$

(iv) $D = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$

11. Find the adjoint of each of the following matrices:

(i) $\begin{pmatrix} 1 & -2 \\ -3 & 4 \end{pmatrix}$

(ii) $\begin{pmatrix} 4 & 6 \\ -2 & -3 \end{pmatrix}$

(iii) $\begin{pmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 11 & 2 & 10 \end{pmatrix}$

(iv) $\begin{pmatrix} 4 & -2 & -1 \\ 1 & 10 & -7 \\ 2 & -4 & 1 \end{pmatrix}$

12. Find A Adj. A for the matrix

$$A = \begin{pmatrix} 1 & -2 & 3 \\ 0 & 2 & -1 \\ -4 & 5 & 2 \end{pmatrix}$$

13. For the square matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{pmatrix}$

Prove that A. Adj. A = |A|. I_3

14. Find the inverse of each of the following matrices:

(i) A = $\begin{pmatrix} 3 & 8 \\ 2 & 1 \end{pmatrix}$ (ii) B = $\begin{pmatrix} 3 & 1 \\ 4 & 0 \end{pmatrix}$ (iii) C = $\begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix}$

15. Find the inverse of each of the following matrices (if it exists)

(i) $\begin{pmatrix} 1 & -1 & 2 \\ -1 & 1 & -1 \\ 1 & -2 & 1 \end{pmatrix}$ (ii) $\begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$ (iii) $\begin{pmatrix} 3 & 2 & 7 \\ 4 & -3 & -2 \\ 5 & 9 & 23 \end{pmatrix}$

16. Find the inverse of the matrix A = $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and how that $AA^{-1} = A^{-1} = A = 1$

17. If A = $\begin{pmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$, find A^2 and A^{-1}, and how that $A^2 = A^{-1}$.

18. Let the matrix A be given by A = $\begin{pmatrix} -1 & 1 \\ 2 & -2 \end{pmatrix}$, verify that $A^2 + 3A + 4 = 0$. Also obtain the inverse of A.

(Unit- 2)

Calculus - I

1. Write short notes on the following :

(1) function Even function, (ii) Odd function, (iii) Periodic function, (iv) Composite f on, (v) Algebraic function, (vi) Transcendental function, and (vii) Parametric function.

2 Explain the following functions along with their notations : .

(i) Exponential function, (ii) Logarithmic function, (iii) Trigonometric function, (iv) Polynomial function, (v) Linear function, (vi) Quadratic function, (vii) Cubic function and (viii) Logistic function.

3. From the following point out which are the functions from A to A and which are not

(i) y = x, (ii) $\sqrt{x}$, (iii) y = {(1,1), (2,2), (7,7)}, (iv) |x| + y = 0, (v) y = $\sqrt{5+2x^2}$, (vi) y = $\frac{3-x}{x-3}$

4. Find fog for each of the following function

(i) $f(x) = \mathrm{x}^8, g(x) = 2x^2 + 1$

(ii) $f(x) = x, g(x) = \frac{1}{x},\ x \neq 0$

(iii) (i) $f(x) = \mathrm{x}^2,\ g(x) = (x+1)$

5. Find gof for each of the following:

(i) $f(x) = \mathrm{x}^2 +\ 1,\ g(x) = \frac{1}{x+1}$

(ii) $f(x) = \sqrt{\mathrm{x}}, g(x)\ 2x^2 + 1$

6. Point out which of the following functions are monotonic and which are not :

(i) y= $+\sqrt{x^2} - 2, x\ \geq 3$

(ii) y = |x|

(iii) y = [x-1]

(iv) y = x^3

(v) y = $\frac{5}{4}$ x

7. Given, A = {1, 2, 3, 4}, and B = {1........... 10}, find the (i) Domain, (ii) Codomain, (iii) Range, (iv) Image, and (v) Pre-image of the function, f: A → B given $f(x)$ = 2x -1.

8. Find the inverse of each of the following function if exists :

(i) $f(x) =\ x^4 +\ 1$, (ii) $f(x) = |\mathrm{x} - 3| + 1$, (iii) $f(x) = +\sqrt{x}, xy$, (iv) $f(x) = 5x^3$,

(v) $f(x) = 2\mathrm{x} + 3$, (vi) $f(x) - \frac{2}{3}\sqrt{9 - \mathrm{x}^2} -\ 3\ \leq \mathrm{x}\ \leq 3$

9. From the following series of functions and inverse functions, arrange the pairs of functions and their respective inverse functions:

Functions	***Inverse functions***
(i) x = y + 5	(i) y = x^2
(ii) x = $\frac{dy-b}{a-cy}$	(ii) y = $\frac{ax-b}{cx+d}$
(iii) x = $\pm\sqrt{y}$	(iii) y = $x-5$

10. Show the function, $f(x)$ defined as $f(x)$ = $\frac{(x+2)(x+1)}{(x+1)}$ = 3, is continous at x =1.

11. Find the derivatives of each of the following function:

(i) x.e^x (ii) $x^2.log\ x$ (iii) $4^x.\ x^4$

(iv) $10^x.x^{16}$ (v) $\sqrt{x}.e^{3x}$ (vi) $x^3.e^{2x}.\log x$

(vii) (2x-3) (4x-5) (viii) $(x^2 - 2)\ (x^3 + 7)$

12. Differentiate each of the following function with respect to x:

(i) $\frac{x^2}{e^x}$ (ii) $\frac{\log.x}{x^3}$ (iii) $\frac{3}{1-5x}$

(iv) $\frac{x^2+1}{x^2-1}$ (v) $\frac{5-4x}{5+4x}$ (vi) $\frac{x^2-1}{x^2+7x+1}$

13. Find the differential coefficient of y with respect to x, when

(i) y= $(x^2+1)\ (3x^2 - 2x)^3$ (ii) y = $\log a\ (x^2+1)$

14. Show that $x^5 - 5x^4 + 5x^3 - 1$ has a maximum value when x = 1, and x = 0, and a minimum value when x = 3.

15. State whether y = $x^3 - 6x^2 + 15$ has a maximum or minimum value. Find the same.

16. Prove that $x^3 - 3x^2 + 3x + 7$, has neither maximum nor minimum.

17. Find the maximum and minimum values of $2x^3 - 3x^2 - 36x + 10$

(Unit- 3)

Calculus - II

1. Integrate the following with respect to x :

(i) x^5, (ii) $3x^5$, (iii) x^{-4}, (iv) 8, (v) 3-2x, (vi) (x+1) (x+2), (vii) $2x^2 - x - 1$, (viii) $(x+1)^2$, (ix) $(x+1)^3$, (x) $3x^2 - 2x + 9$ (xi) $\frac{5}{x^3}$, (xii) $(ax + b)^{-2}$, (xiii) $\frac{x^3+x^2-1}{x^2}$

2. Evaluate the following integarls :

(i) $\int (x + \frac{1}{x})\, dx$,

(ii) $\int \frac{2x-3}{6x^2} dx$,

(iii) $\int \frac{2x^3-5x^2+7}{x} dx$,

(iv) $\int (x^{\frac{3}{2}} + 2x^2 - 4)dx$,

(v) $\int (2x + \frac{3}{x})^2 \ dx$,

(vi) $\int [2x^{-\frac{1}{2}} + \sqrt{x} - \sqrt[3]{x} + (2x+3)^2 dx]$,

(vii) $\int \left(4e^x + \frac{5}{x} - \frac{1}{x^2}\right) dx$

(viii) $\int (10^{2x} + e^{-0.5x} + \frac{1}{x^3})\, dx$

(ix) $\int \left(e^{x/2} - 7x^{3/2} + 5e^{2x}\right)dx$

(x) $\int 3 \times 2^x\, dx$

(xi) $\int ae^{-mt}\, dt.$

3. Evaluate the following:

(i)$\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) dx$ (ii) $\int \frac{ax^3+bx^2+cx+d}{x} dx$, (iii) $\int \frac{4x^6+3x^5+2x^4+x^3+x^2+1}{x^3} dx$,

(iv) $\int 2^x + \frac{1}{2}e^{-x} + \frac{4}{x} - \frac{1}{\sqrt[3]{x}} - \ dx.$

4. The demand function for an article is p = 20-2x-x^2 . Determine the consumer's surplus when demand is 3.

5. For a monopolistic demand function p = $(6\text{- x})^2$ and marginal cost = 14 + x. Determine the consumer's surplus.

6. The supply law for a commodity is p = 4+x. Find the producer's surplus when 12 units of goods are sold.

7. The demand and supply function for an article respectively are:
P_d = 18 – 2x - x^2, and P_8 = 2x – 3. Determine the consumer's surplus and producer's surplus at the equilibrium price.

8. If the demand and supply equations for an article respectively are:
P_d = 56-x^2 and P_s = 8 + $\frac{x^2}{3}$, then find the consumer's surplus and producer's surplus.

9. The demand and supply laws under perfect competition respectively are:
P_d = 16-x^2, and P_s = $2x^2 + 4$. Determine the market price, consumer's surplus and producer's surplus.

10. The quantity demanded, and the corresponding price under perfect competition are determined by the demand and supply techniques q = 36, and q = 6+$\frac{P^2}{4}$ respectively. Find the corresponding consumer's surplus and producer's surplus.

11. The demand function of a commodity is p - 10 e^{-x} = 0. Find the consumer's surplus when the market price p = 1.

12. A monopolistic demand function is x = 210 – 3p where x is the quantity demanded when price is Rs p per unit. With the average cost function AC = x + 6 + $\frac{10}{2}$ find the consumer's surplus at the price which the monopolist will like to fix.

13. Explain the concept of the learning curve and state the areas where such a curve can be used.

14. A firm's learning curve after producing 100 units is given by f(x)=2400$x^{-0.5}$, which is the rate of labour hour required to produce the xth unit. Find the hours needed to produce an additional 800 units.

15. A firm's monthly sales are Rs. 50,000/- which are expected to rise by 10% per month. What will be the monthly sales in 10 months?

16. After producing 36 units of commodity, a firm has a learning curve $f(x)$ = 1000$x^{-0.5}$. Determine the labour hour (H) required to produce the next 28 units.

17. If in the manufacture of an article, a firm has spent 2000 man hours for its first unit, then how many hours it would take for 200th unit to produce ? Take the learning curve f (x)= A$x^{-0.3220}$at 80%.

15. Explain the different, natures of the products and state how they are determined with the help of the partial derivatives.

16. Determine (i) time path of capital (K) and (ii) the amount of capital formation during the time interval [1,3]
where, (i) rate of investment is I (t) = 12 $t^{1/3}$, and (ii) the initial stock is K(0) = 25.

17, If the net investment flow is described by the equation 1(t) = 6$t^{1/5}$, and that K(0) = 20, then find (i) the time path of capital (K), and (ii) the amount of capital formation during the time interval [t-1, t]

18. If the net investment flow is denoted by the function I(t) = $3t^{1/2}$, and that the capital stock at the time t = 0 isK_0, then determine the time path of capital K.

19. Find the capital formation on the time period of 5 years, and during the last year of the plan *i.e.* 5th Year when the investment flow function is I(t) = $3t^{1/2}$ crores of rupees per year. Also, find the capital growth equation from the given rate of capital formation if the initial stock of the capital is 20 crores of rupees.

(Unit- 4)

Mathematics of Finance

1. At what rate percent will Rs. 4, 500 become Rs. 6,120 in 4 years ?

2. If Rs. 1,200 amounts to Rs. 1,500 in 5 years, find the rate of simple interest.

3. A sum of Rs. 550 was lent out for 2 years at simple interest. The lender got a total amount of Rs. 638. Find the rate of interest per cent per annum.

4. If the simple interest on a sum equals to $\frac{1}{10}$ of itself in 4 years, find the rate percent?

5. Determine the time in which a sum of Rs. 2,400 fetches Rs. 120 as S.I. at 5% p.a.

6. In what time Rs. 1,250 will amount to Rs. 1,400 at 6% p.a. S.I. ?

7. In what time a sum will be $\frac{29}{10}$ times of itself at 5% p.a. S.I. ?

8. Digvijay borrowed Rs. 1,500 at 5% p.a. After some time he returned Rs. 1,800 and cleared the account. For how much time did he keep the money ?

9. In what time Rs. 5,000 will yield 1,100 at $5\frac{1}{2}$ %.

10. An asset yields an annual income of Rs. 25,000 for 5 years. Find the value of the asset and the rate of income.

11. Determine the rate of interest at which a sum of Rs. 3,200 amounts to Rs. 5529.60 in 3 Years?

12. Find the rate of compound interest at which Rs. 10,000 amounts to Rs. 12882.25 in 2 years.

13. Find the compound interest on Rs. 12,000 for $1\frac{1}{2}$ years, at 16% p.a. interest being compounded quarterly.

14. Find the amount of Rs. 6,000 in 2 years at 8% p.a., interest being compounded half yearly.

15. lf, the annual increase in the population of a state is 25 per thousand and the present number of its inhabitants is 26,24,000, what will be the population in 3 years hence ? What was it, a year ago ?

16. A machine costing Rs. 5,810 depreciates @ 10% of its value at the beginning of a year. If it fetches a scrap value of Rs. 2,250, find the number of years for which the machine was in use.

17. Find the difference between the Compound interest and the Simple interest on

(i) 5,000 for 4 years at 5% p.a.
(ii) 6,000 for 3 years at 6% p.a. compounded half yearly.
(iii) 1,000 for 2 years at 4% p.a. payable quarterly.

18. Find the number of years in which a sum

(i) doubles itself at 4% p.a. compound.
(ii) trebles itself at 5% p.a. compound.
(iii) quadruples itself which doubles in 4 years.

(Unit- 5)

Liner Programming

1. A firm produces two types of products P & Q and sells them at a profit of Rs. 2 and Rs. 3 respectively. Each product passes through two machines, R and S. P requires one minute of processing time on R and two minutes on S. Q requires one minute on R and one minute on S. The machine, R is available for not more than 6 hours 40 minutes while the machine S is available for 10 hours during any working day.

Formulate the above as a linear programming problem.

2. A firm produces two types of mats. Each mat of the first type needs twice as much labour as the second type. If all the mats are of the second type only, the firm can produce a total of 500 units mats a day. The market limits daily sales of the first and second type to 150 to 250 respectively. Assuming that the profit per mat are Rs. 8 and Rs. 5 respectively for the two types, formulate the problem as a linear programming model to determine the number of mats to be produced of each type so as to maximize the profit.

3. A firm with 100 acres of land can sell all tomatoes, lettuce or radishes that it can raise. The sales prices are Rs. 1.00 per kg. of tomato, Rs. 0.75 per head of lettuce, and Rs. 2.00 per kg. of radishes. The average-yield per acre are : 2000 kgs. of tomatoes. 3000 heads of lettuce and 1000 kgs of radishes. Fertilizer is available at Rs. 0.50 per kg and the amount required per acre are 1000 kgs each for tomatoes and lettuce and 50 kgs for radishes. Labour required for sowing, cultivating, and harvesting per acre is 5 man-days for tomatoes, and radishes and 6 man-days for lettuce. A total of 400 man- days of labour are available at Rs. 20 per man-day. Formulate the problem as a linear programming model to maximize the firm's total profit.

4. A drug manufacturing company proposed to prepare a production plan for medicines, X and Y. There are sufficient ingredients available to make 20000 bottles of the medicine X and 40000 bottles of the medicine Y but there are only 45000 bottles into which either of the medicines can be filled in. It takes 3 hours to prepare sufficient material to fill 1000 bottles of the medicine X and one hour to prepare enough materials to fill 1000 bottles of the medicine Y, and there are only 66 hours available for this operation. The profit is Rs. 8 per bottle for the medicine X and Rs. 7 per bottle for the medicine Y.

From the above

(a) Formulate the problem as a LPP and

(b) determine how the Company would schedule its productions in order to maximize its profit.

5. A firm produces two types of toy, A and B. A takes twice as much as to produce as B and the firm has time to produce a maximum of 2000 per day. The supply of plastic is sufficient to produce 1500 toys per day (both A and B). B requires a fancy dress of which there are only 600 per day available. If the firm makes a profit of Rs. 3 and Rs. 5 per toy respectively as A and B, then how many of each toy should be produced per day in order to maximize the total profit. Solve this problem by graphic method.

6. In a chemical Industry, two products, P and Q are produced through two operations. The production of Q also results in a by-product, R. The product, P is sold at a profit of Rs. 3 per unit and the product Q at a profit of Rs. 8 per unit. The by- product R can be sold at a profit of Rs. 2 per unit, but it can not be sold as the destruction cost is Re. 1 per unit. Forecasts indicate that only 5 units of R can be sold. The industry gets 3 units of R for each unit of P and Q produced. Forecasts show that they can sell all the units of P and Q produced. The production times are 3

hours per unit of P on operation I and II respectively and 4 hours and 5 hours per unit of Q on operation I and II respectively. Because, the product, R results from Q, no time is used in producing R. The available times are 18 and 21 hours of operation I and II respectively. The industry asks how much of P and Q should be produced keeping R in mind to make the highest profit. Formulate an L.P. model for the above problem and solve the same by the graphic method.

7. A factory produces two articles, C and D. For C, machine hours required are 1.5 hours and a craftsman has to work for 2 hours. For D, machine hours required are 2.5 hours, and a craftsman has to work for 1.5 hours. In a week the factory can avail of 80 hours of machine time and 70 hours of craftsman time. The profit on each article of C is Rs. 5 and that on each article of D is Rs. 4. If all the articles produced can be sold off, find graphically how many of each kind should be produced to earn the maximum profit per week ?

8. Define a linear programming problem. State the different types of linear programming problems and briefly point out the limitations of an L.P.P.

9. Explain briefly the formulation procedure of a linear programming problem.

10. What are the three major problems that can be solved using the linear programming techniques. Discuss each of them briefly.

11. What do you mean by duality in the linear programming. Distinguish a Dual problem from a primal one.

12. Explain briefly the following concepts of a linear programming problem :

(a) Objective function. (b) Constraint functions. (c) Non-negativity condition.

13. Write short notes on :

(i) Degeneracy (ii) Slack Variable (iii) Surplus variable (iv) Artificial variable. (v) Limitation of L.P.P.

14. Solve the following L.P.P. by graphic method :

Maximize $Z = 5X_1 + 3X_2$
Subject to $3X_1 + 5X_2 \leq 15$
$5X_1 + 2X_2 \leq 10$
$X1, X2 \geq 0$

15. A wholesale dealer deals in two kinds of mixtures, X and Y. Each of the mixture X contains 60 grams of almonds, 30 gms. of cashewnuts, and 30 gms of hazel nuts. Each kg. of the mixture B contains 30 gms of almonds, 60 gms of cashewnuts and 180 gms of hazel nuts. The remainder of both the mixtures is peanuts. The dealer is contemplating to use the mixture X and Y to make a bag which will contain at least 240 gms of almonds 300 gms of cashewnuts and 540 gms of hazel nuts. Mixture X costs Rs. 8 per kg. and mixture Y costs Rs. 12 per kg. Assuming that the mixtures X and Y are uniform, determine, how many kgs of each mixture should be used to minimize the cost of the bag.
If there is no market for the nut mixture priced at Rs. 15 per kg., will you advice the dealer to introduce the type of mixtures he is contemplating. Use graphic method to give your answers.

16. A diet for a patient should contain at least 4000 units of vitamins, 50 units of minerals and 1400 units of calories. Two foods, P and Q are available at a cost of Rs. 4 and Rs. 3 per unit respectively. If one unit of P contains 200 units of vitamins, 1 unit of minerals and 40 units of calories and one unit of Q contains 100 units of vitamins, 2 units of minerals and 40 units of calories, fmd by simplex method what combination of food be used to have the least cost.

17. A firm must produce 200 kgs. of a mixture consisting of the ingredients, P and Q which cost 3 and Rs. 8 per kg respectively. Not more than 80 kgs. of P can be used and at least 60 kgs. of Q must be used. Find by the simplex method how much of each ingredient should be used in order to minimise the cost.

18. A marketing manager wishes to allocate his annual advertisement expenditure of 20,000 in the two media, A and B. The unit cost of a message in the media, A is Rs. 1000, and that of B is Rs. 1500. The Media A is a monthly magazine and not more than one insertion is desired in one issue. At least 5 messages should appear in the media B. The expected effective audience for unit messages in the media A is 40000 and for the media B is Rs. 55000.
From the above information formulate an L.P. model and solve the problem by the simplex method for maximizing the total effective audience.